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Bow–arrow interaction in archery

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A mathematical model of the flight of the arrow during its discharge from a bow was proposed by Pękalski (1990). His description of the model was incomplete. In this paper, I give a full description of the model. Furthermore, I propose some improvements that make his model more consistent with reality. One achievement is the modelling of contact of the arrow and grip; the pressure button is modelled as a unilateral elastic support. The acceleration force acting upon the arrow during the launch is predicted by an advanced mathematical model of bow dynamics. There is a satisfactory conformity of the simulation and experimental results. The new model predicts that the arrow leaves the pressure button before it leaves the string, as reported previously. The ability to model arrow dynamics can be used to improve the adjustment of the bow–arrow system for optimal performance.

Keywords: 'archer's paradox', archery, arrow motion, shooting, simulation.

Introduction

Pękalski (1990) proposed new methods and research techniques in archery. He discussed the following topics:

- A mathematical model of an arrow's movement during its interaction with a bow. The governing equation is a linear fourth-order parabolic partial differential equation with boundary and initial conditions.
- A mechanical model of an archer–bow–arrow system; this is a mechanical shooting machine that provides reproducibility.
- Pękalski filmed (1000–2500 Hz high-speed 16-mm cine film) the arrow release by a female member of the Polish National team and an arrow released from the shooting machine. Figure 1 shows the arrow's transverse motions taken from the film made with the camera viewing the archer from above.

The aims of this paper are three-fold. First, I comment on the mathematical model of an arrow's movement during its interaction with a bow as described by Pękalski (1987, 1990). Pękalski made simplifying assumptions to solve the governing equations (a linear partial differential equation) using Fourier's method and Krilov's function base. Secondly, to provide insight into these assumptions, his theory is discussed in its entirety; in fact, his analysis was the incentive for reconsidering the problem. I present a more accurate model and focus on the biomechanical aspects and the use of the knowledge gained in the sport of archery; the mathematical aspects are presented elsewhere (Kooi and Sparenberg, 1997). The system obtained is non-linear, and has to be solved numerically. A finite difference technique was used to solve the non-linear partial differential equations with initial and (moving) boundary conditions. The third aim is to compare the predictions of the model with results reported in the literature.

All modern bows have an arrow rest on which the arrow is supported vertically; the arrow is supported horizontally in a unilateral fashion by a pressure button (shock absorber) with a built-in spring (Bolnick *et al.*, 1993). Gallozzi *et al.* (1987) and Leonardi *et al.* (1987) showed experimentally that the arrow is in contact with the grip for a period that is shorter than the duration of the contact between the arrow and the string. Hence, after some moment, the arrow is free from the grip while it is still being accelerated by the string until it separates from the string. This phenomenon is also clearly seen in recent videos (Sanchez, 1989, filming Olympic gold medallist Jay Barrs; Rabska and van Otteren, 1991). This experimental observation is also predicted by the new model.

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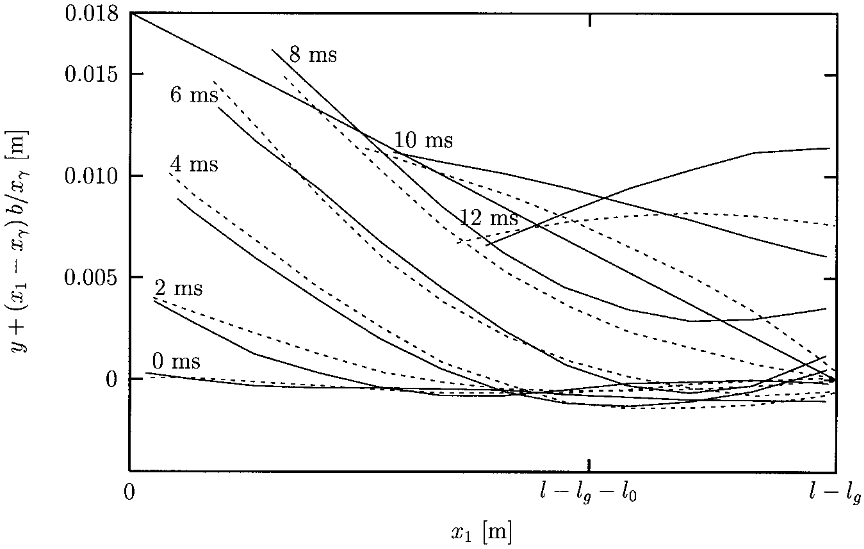

Figure 1 Curves of an arrow's deflection on the basis of experimental data (– – –) and on the basis of Pękalski's mathematical model (——), $y + (x_1 - x_\gamma)b/x_\gamma$, every 2 ms after release (after Pękalski, 1990). The shape of the arrow is shown as a coordinate system fixed to the bow; the arrow when fully drawn is on the horizontal axis and the position of the grip of the bow is point $(l - l_g, 0)$. The solid line between points $(0, b)$ and $(l - l_g, 0)$ indicates the median plane of the bow.

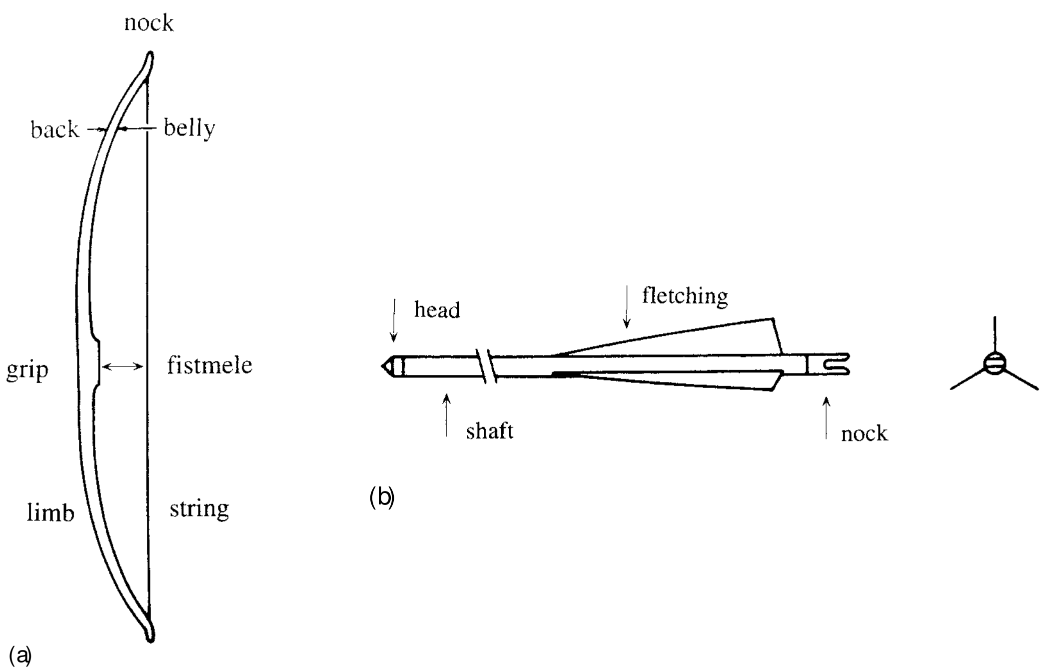

Figure 2 Anatomy of the bow (a) and the arrow (b).

The simulation results provide an understanding of arrow dynamics. When detailed experimental data are extracted from video analysis, the model can be used to improve the adjustment of the bow–arrow system for optimal performance.

In essence, the bow proper consists of two elastic limbs, often separated by a rigid middle part, the grip. The bow is braced by fastening a rather stiff string between both ends of the limbs (Fig. 2). The anatomy of the arrow is also shown in Fig. 2. The arrow consists of

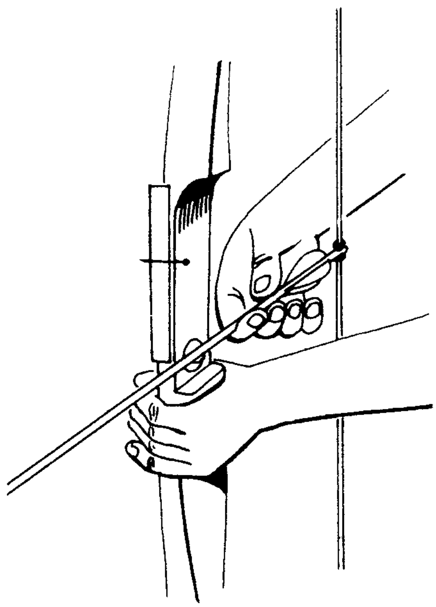

Figure 3 An arrow set on the string (after Baier *et al.*, 1976). The nock is provided with a groove in which the string sticks slightly when the arrow is set on the string. The arrow is supported vertically by the arrow rest fixed to the bow grip and horizontally in a unilateral fashion by a pressure button with a built-in spring (not shown).

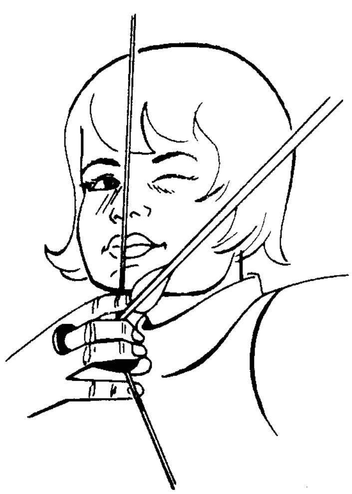

Figure 4 With the 'Mediterranean release', the first three fingers draw the string, while the engaged arrow rests between the first and the second fingers. The string is located over the second interphalangeal joints. In full draw, the second joint of the index finger of the drawing hand touches below the centre of the chin (after Baier *et al.*, 1976).

a shaft, arrowhead at the front-end, fletching and nock at the rear-end. The nock is provided with a groove in which the string sticks slightly when the arrow is placed on the string (nock) (Fig. 3). The archer then extends the bow arm (extend) and pulls the bow from the braced position into full draw (draw). We assume the so-called 'Mediterranean release'. The first three fingers draw the string, while the engaged arrow rests between the first and second fingers (Fig. 4). The string is located over the second interphalangeal joints. In full draw, the index finger of the drawing hand touches below the centre of the chin (anchor). This completes the static action in which potential energy is stored in the elastic parts of the bow.

After aiming (hold, aim), the arrow is loosed by extending the pull fingers (release). The force in the string accelerates the arrow and transfers part of the available energy as kinetic energy to the arrow. Meanwhile, the bow is held in place and the archer feels a recoil force in the bow hand (afterhold). After the arrow has left the string, the bow returns to the braced position because of damping. Klopsteg (1992) dealt with the physics of bows and arrows and Kooi (1991) proposed a mathematical model of the bow. Readers are referred to Axford (1995) for a detailed description of the interrelationship between the anatomy of the human body and the anatomy of the bow and arrow.

The following sequence of shooting motions is identified (Baier *et al.*, 1976; Bolnick *et al.*, 1993): stand, nock, extend, draw, anchor, hold, aim, release and afterhold. Leroyer *et al.* (1993) mentioned three phases only: stance (stand), arming (nock, extend, draw, anchor) and the sighting (hold, aim). They analysed the displacement pull-hand measurement during the final push–pull phase of the shoot. Pękalski (1990) divided the ballistics of the arrow into two phases:

- *Phase 1, internal ballistics*: the interaction between the arrow and the archer–bow system until the arrow leaves the string.
- *Phase 2, external ballistics*: this phase lasts from the end of phase 1 until the arrow hits the target.

This paper deals with phase 1 extended to the moment the arrow passes the grip to study the so-called 'archer's paradox' (Baier *et al.*, 1976; Klopsteg, 1992); the arrow 'oscillates its way' past the bow without the rear-end 'slapping' against the grip of the bow.

With respect to the dynamics of the arrow, the following dimensions of the arrow are important, using the notation introduced by Pękalski (1990). The total mass of the arrow (m_s) is the sum of the mass of the arrow shaft (m_k), arrowhead (m_g), fletching (m_p) and nock (m_n). The length of the arrow is denoted by l. It is

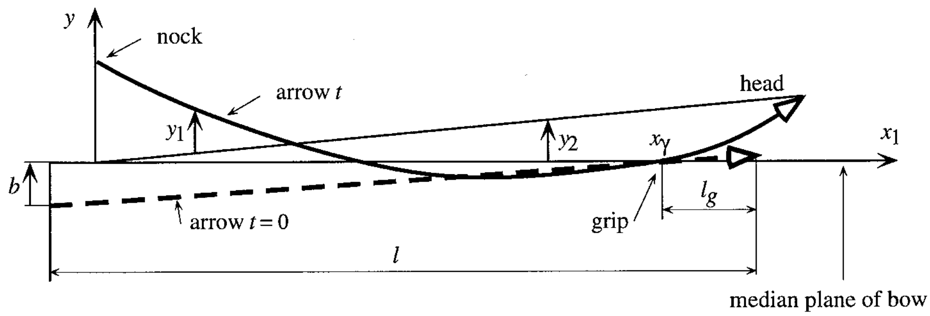

Figure 5 The solid line shows the shape of the arrow and the dashed line is the shape at $t = 0$, which is out of the median plane of the bow. The displacement out of the median plane is $y = y_1 + y_2$.

Table 1 Values for the parameters of the Easton 1714X7 (Aluminum 7178) arrow (after Pękalski, 1987)[a]

Parameter	Description	Value
Arrow		
l	length	0.67 m
d	external diameter	6.75 mm
g	wall thickness	0.356 mm
ρA	mass per unit length	0.02 $kg \cdot m^{-1}$
EJ	flexural rigidity	0.209 $N \cdot m^{-2}$
m_s	mass of arrow	0.0185 kg
m_k	mass of shaft	0.0135 kg
m_g	mass of head	0.004 kg
m_p	mass of fletching	0.001 kg
m_n	mass of nock	0.004 kg
Bow		
k_{xs}	bow stiffness	342 $N \cdot m^{-1}$
k_{ys}	transverse stiffness	270 $N \cdot m^{-1}$
l_0	initial drawing length	0.225 m
l_n	drawing length	0.645 m
$F_n(l_n)$	drawing force	143 N
Arrow and bow		
k_g	sprint constant of pressure button	622 $N \cdot m^{-1}$
l_g	distance between head and grip	0.025 m

[a] The bow is a Hoyt pro-medallist T/D, 1.68 m (66 inches), 15.5 kg (34 lb). For the standard arrow–bow combination, $\eta_x = \eta_y = 0.75$.

measured from the rear-end, the nock, to the fore-end, the head (Fig. 5). The position of the arrow with respect to the bow is determined by the distance, l_g, between the arrowhead and the grip of the bow in full draw. The distance between the arrow nock and the grip is denoted by l_n. In the braced position, this distance is denoted by l_0. The dimensions of a modern bow–arrow system are given in Table 1.

Pękalski's model

Pękalski modelled the movements of the arrow during its launch. While being accelerated, the arrow vibrates in the horizontal plane.

Forward movement

The draw-force, F_n, is assumed to be proportional to the draw-length, l_n, minus the brace height, l_0:

$$F_n = k_{xs}(l_n - l_0) \tag{1}$$

The acceleration force is assumed to be:

$$E_n = \eta_x k_{xs}(l_n - l_0) \tag{2}$$

That is, the bow is modelled as a simple linear spring with stiffness k_{xs} and efficiency η_x. The movement of the arrow towards the target during the release from the bow, follows from this force when the arrow is approximated by a particle with mass m_s. Consequently, the acceleration of the arrow equals E_n/m_s.

Transverse movement in the horizontal plane

The transverse movement of the arrow in the horizontal plane (vertical movements are neglected) has two components:

$$y(x_1,t_1) = y_1(x_1,t_1) + y_2(x_1,t_1) \tag{3}$$

where x_1 is the length coordinate along the arrow measured from the rear-end, $y_1(x_1,t_1)$ is a vibrational movement and $y_2(x_1,t_1)$ is a rotational motion of the arrow around the nock (Fig. 5). The second independent variable, t_1, denotes time. The arrow is placed in a Cartesian coordinate system (x_1,y), the origin moving along the median plane of the bow.

The vibrating movement satisfies the well-known 'beam equation':

$$EJ\frac{\partial^4 y_1}{\partial x_1^4}(x_1,t_1) + A\rho\frac{\partial^2 y_1}{\partial t^2}(x_1,t_1) = 0 \quad (4)$$

The so-called Euler Bernoulli equation is assumed. Then the curvature $\partial^2 y_1/\partial x_1^2$ is proportional to the bending moment, the proportionality constant being the flexural rigidity (bending stiffness), denoted by EJ, where E is Young's modulus of the material and J is the second area moment of inertia of the cross-section with respect to the neutral axis of the arrow. The shear force equals $-EJ\partial^3 y_1/\partial x_1^3$. Equation (4) then follows from the equations of motion of a typical element of the beam, where ρ is the density and A is the cross-sectional area (Timoshenko *et al.*, 1974). For a modern tubular arrow shaft, the area A and the second moment of inertia J are given by:

$$\begin{aligned} A &= \pi(d^2 - (d-2g)^2)/4 \\ J &= \pi(d^4 - (d-2g)^4)/64 \end{aligned} \quad (5)$$

where d and g are the external diameter and the shaft wall thickness, respectively.

The boundary conditions at the nock, $x_1 = 0$, are:

$$\begin{aligned} &\frac{\partial^2 y_1}{\partial x_1^2}(0,t_1) = 0 \\ &EJ\frac{\partial^3 y_1}{\partial x_1^3}(0,t_1) + \eta_b k_{ys} y_1(0,t_1) = 0 \end{aligned} \quad (6)$$

where k_{ys} is the static transverse elasticity of the bow and η_b is the associated efficiency. The first equation means that the bending moment is zero at the nock. The second states that the shear force equals the force in the spring, which represents the transverse elasticity of the bow.

The boundary conditions at the arrowhead, $x_1 = l$, are:

$$\begin{aligned} &\frac{\partial^2 y_1}{\partial x_1^2}(l,t_1) = 0 \\ &y_1(l,t_1) = 0 \end{aligned} \quad (7)$$

The first equation means that the moment at the arrowhead is zero and the second equation means that the transverse displacement is zero. That is, the arrowhead is placed in a hinge-like joint.

The initial conditions for $y_1(x_1,t_1)$ at $t_1 = 0$ are:

$$\begin{aligned} &y_1(x_1,0) = \frac{b}{l}(x_1 - l) \\ &\frac{\partial y_1}{\partial t_1}(x_1,0) = 0 \end{aligned} \quad (8)$$

The nock is a distance b out of the median plane of the bow, for a right-handed archer to the right. The deflection b of the nock is a parameter that depends on the archer's technique.

The second movement, $y_2(x_1,t_1)$, is the rotation of the arrow such that $y_2(0,t_1) = 0$. Let t_f denote the instant the arrow becomes free from the grip, then $y_2(x_1,t_1)$ is determined during $0 \le t_1 \le t_f$ by the requirement that the arrow remains in contact with the grip where the x_1 coordinate is denoted by $x_\gamma(t_1)$. Thus:

$$y(x_1,t_1) = y_1(x_1,t_1) - \frac{x_1}{x_\gamma(t_1)} y_1(x_\gamma(t_1),t_1) \quad (9)$$

where $y(x_\gamma(t_1),t_1) = 0$, since the arrow is in contact with the grip. The function $x_\gamma(t_1)$ follows from the forward movement of the arrow:

$$\begin{aligned} \ddot{x}_\gamma(t_1) &= -E_n(t_1)/m_s \\ \dot{x}_\gamma(0) &= 0 \\ x_\gamma(0) &= l - l_g \end{aligned} \quad (10)$$

The acceleration of the point of contact on the grip equals the acceleration of the arrow.

In Pękalski's theory, t_f is determined by the moment at which the transverse velocity of the arrowhead, $V_y(t_1) = \partial y/\partial t_1(l,t_1)$, is at its maximum. For $t_1 \ge t_f$, Pękalski assumed that the velocity, V_{ymax}, remains unaltered. Then, the following total movement $y(x_1,t_1)$ occurs after the arrow loses contact with the grip:

$$y(x_1,t_1) = y_1(x_1,t_1) + \frac{x_1}{l}(y(l,t_f) + (t_1 - t_f)V_{ymax}) \quad (11)$$

Pękalski solved the linear partial differential equation for the variable $y_1(x_1,t_1)$ (equation 4), with the boundary conditions (6) and (7), together with the initial condition (8), by means of the Krilov function technique. Substitution of this solution in equation (9) for $0 \le t_1 \le t_f$, and equation (11) for $t_1 \ge t_f$, yields the shape of the arrow.

Kooi–Sparenberg model

In this section, the model proposed by Kooi and Sparenberg (1997) is described. The deviations from Pękalski's model are emphasized and the reasons for the improvements are given.

Forward movement

The simple linear model for the bow Pękalski used is replaced by the model described in Kooi (1991). Figure 6

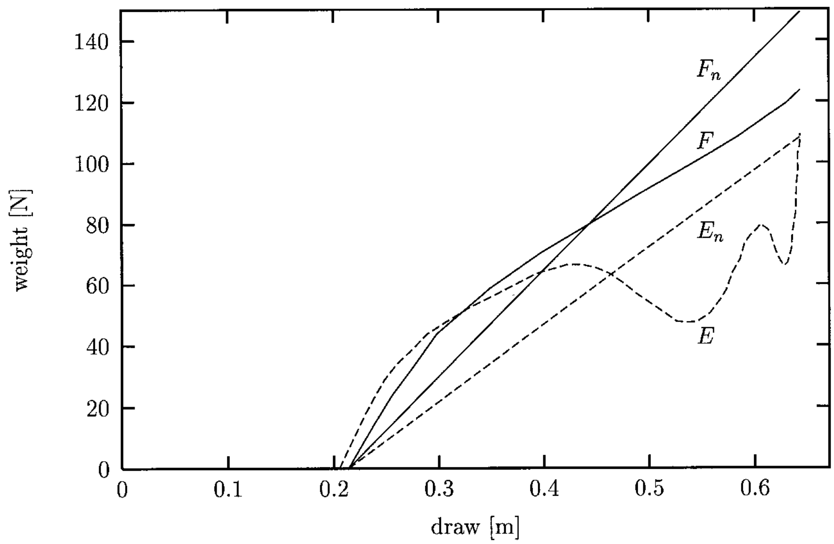

Figure 6 Calculated static force draw curve, F, and dynamic force draw curve, E, for a modern working-recurve bow. The static force draw curve, F_n, and the dynamic force draw curve, E_n, are for the associated linear-spring bow used by Pękalski (1990).

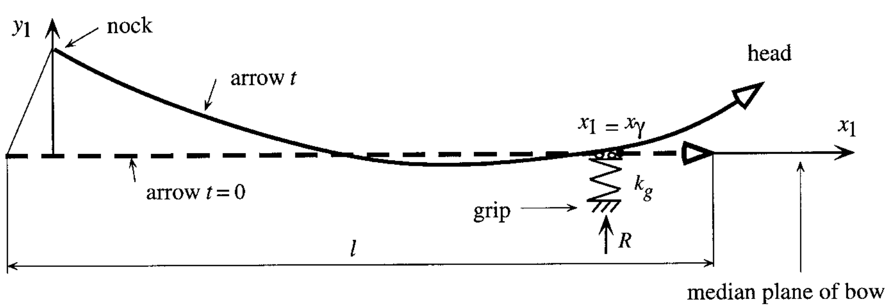

Figure 7 The solid line shows the shape of the arrow when it clears the finger tab. The dashed line shows the shape at $t = 0$, which is in the median plane of the bow.

gives the calculated static force draw curve, denoted by F, of a modern competition bow. It is the force required to draw an arrow to a specified draw length. This curve is approximated by that of the linear spring, F_n, which is proportional to the actual draw length, so that the areas below both curves (representing the available energy to be transferred to the arrow) are equal. Pękalski assumed the dynamic force draw curve (the non-linear acceleration force, E, acting upon the arrow) to be the efficiency η_k times the values of the static force draw curve or $E_n = \eta_k F_n$. In Fig. 6 we show also the calculated dynamic force draw curve for a modern bow. The mathematical model and the applied numerical technique are described elsewhere (Kooi, 1991).

Transverse movement in the horizontal plane

In this new model, the displacement $y_1(x_1,t_1)$ of the arrow is again measured from the median plane of the bow (see Fig. 7). The longitudinal force, $H(x_1,t_1)$ (positive for tensile forces), due to the acceleration force is taken into account:

$$EJ\frac{\partial^4 y_1}{\partial x_1^4} - \frac{\partial H \partial y_1/\partial x_1}{\partial x_1} + A\rho\frac{\partial^2 y_1}{\partial t_1^2} = 0 \quad (12)$$

The longitudinal force is given by:

$$H(x_1,t_1) = -\frac{\rho C(l - x_1) + m_g}{m_s} E_n(t_1) \quad (13)$$

One of the main differences between Pękalski's model and the model proposed here is the way the release is modelled. With the 'Mediterranean release', the first three fingers draw the string, located over the second interphalangeal joints. In the model presented here, starting in the median plane, the string slips off the fingertips making the nock of the arrow off-centre to the median plane of the bow (see also Baier *et al.*, 1976; Axford, 1995). In mathematical terms, this means that the path of the nock $y_1(0,t_1)$ is prescribed for the period the fingertips contact the nock. For the length of the contact line, 0.0035 m was used in the longitudinal x_1-direction and 0.00229 m in the transverse y_1-direction. These values were obtained by trial and error to obtain a good correlation with experimental results. These parameters depend on the archer's technique. Figure 7 shows that the arrow is initially in the median plane, so the initial conditions are:

$$\begin{aligned} & y_1(x_1,0) = 0 \\ & \frac{\partial y_1}{\partial t_1}(x_1,0) = 0 \end{aligned} \tag{14}$$

that is, the transverse velocity is zero.

It follows from equations (8) and (6) that, directly after release, the nock in Pękalski's model has a velocity towards the median plane as a result of the transverse elasticity of the bow, because in his model the nock of the arrow starts off-centre ($b \leq 0$). The same happens with the nock in this model when the string slips off the fingertips. Hence, just after release, there are similar motions of the nock in both models. In this model, the artificial initial position of the arrow off-centre, b, equivalent to a bow's torsion around the vertical axis, is not needed.

After the arrow has left the fingertips, the boundary conditions at the place where the nock of the arrow $x_1 = 0$ resemble those in Pękalski's model (equation 6):

$$\begin{aligned} & \frac{\partial^2 y_1}{\partial x_1^2}(0,t_1) = 0 \\ & EJ\frac{\partial^3 y_1}{\partial x_1^3}(0,t_1) - H\frac{\partial y_1}{\partial x_1}(0,t_1) + \eta_b k_{ys} y_1(0,t_1) = 0 \end{aligned} \tag{15}$$

where the mass of the nock m_n is neglected. After the arrow has left the string, the instant denoted by t_l, the last term in the second equation, disappears (that is, the transverse force becomes zero).

Another difference is the modelling of the contact between the arrow and the grip. As in Pękalski (1990), the protrusion of the arrow's rest, denoted by y_p, is zero. In this model, the contact force is taken into account explicitly, making the third-order spatial derivative of the deflections discontinuous at the place of contact, $x_1 = x_\gamma(t_1)$, which is a function of time owing to the forward motion of the arrow in equation (10), where $E_n(t_1)$ is replaced by $E(t_1)$ (see Fig. 6).

The contact force between arrow and grip, $R(t_1)$, is proportional to the discontinuity of the third-order partial derivative:

$$R(t_1) = \lim_{x_1 \downarrow x_\gamma(t_1)} \frac{\partial^3 y_1}{\partial x_1^3}(x_1,t_1) - \lim_{x_1 \uparrow x_\gamma(t_1)} \frac{\partial^3 y_1}{\partial x_1^3}(x_1,t_1) \tag{16}$$

This force, acting in the transverse y_1-direction, is also equal to the force in the spring of the pressure button:

$$\begin{aligned} & R(t_1) = \begin{cases} -k_g y_1(x_\gamma(t_1),t_1) & \text{if } y_1(x_\gamma,t_1) \geq 0 \\ 0 & \text{if } y_1(x_\gamma,t_1) > 0 \end{cases} \\ & t_1 \geq t_f \end{aligned} \tag{17}$$

where k_g is the spring constant of the pressure button. The moment the arrow is in contact with the grip for the last time is denoted by t_f.

The boundary conditions at the tip of the arrow, $x_1 = l$, read:

$$\begin{aligned} & \frac{\partial^2 y_1}{\partial x_1^2}(l,t_1) = 0 \\ & EJ\frac{\partial^3 y_1}{\partial x_1^3}(l,t_1) - H\frac{\partial y_1}{\partial x_1}(l,t_1) - m_g\frac{\partial^2 y_1}{\partial t^2}(l,t_1) = 0 \end{aligned} \tag{18}$$

The mass of the arrowhead, m_g, is taken into account as a point mass.

This completes the description of the mathematical model. In contrast with Pękalski's model, Fourier's method cannot be used to analyse this system of non-linear partial differential equations with boundary and initial conditions (because of the contact problem). Kooi and Sparenberg (1997) proposed a finite difference technique (Mitchell and Griffiths, 1980) to solve the equations numerically; the results presented were calculated using such a method.

Results

In Fig. 1 we give the shapes calculated by Pękalski for a standard arrow shot with a standard bow every 2 ms after release. The parameter settings for the standard arrow and bow are given in Table 1. The solid line between the points $(0,b)$ and $(l - l_g,0)$ indicates the median plane of the bow. Pękalski used a non-linear least-square regression technique to estimate the following parameter values: $\eta_x = 0.76$, $\eta_y = 0.71$ and $b = -0.018$ m.

Figure 8 gives the shapes $y_1(x_1,t_1)$ calculated with the new model for a standard arrow shot with a standard bow every 2 ms after release until the nock passes the grip. Figure 9 gives the shapes of the arrow predicted by our model but now with the acceleration force E of a modern bow instead of the linear one, E_n. The modern bow was also used in the experiments described by Tuijn and Kooi (1992); the measured efficiency was only a few percent below the value predicted by the model.

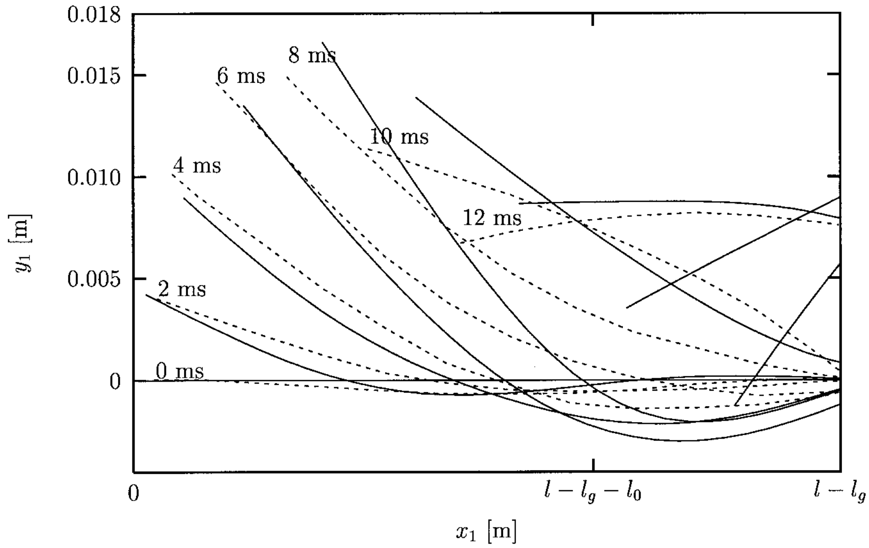

Figure 8 Curves of an arrow's deflection on the basis of experimental data (– – –) (after Pękalski, 1990) and on the basis of the mathematical model (——) every 2 ms after release. The bow is modelled as a linear spring where the drawing force, E_n, is proportional to the actual drawing length (see Fig. 6). The shape of the arrow is shown as a coordinate system fixed to the bow; the arrow when fully drawn, in the median plane of the bow, is on the horizontal axis; the position of the grip of the bow is point $(l - l_g, 0)$.

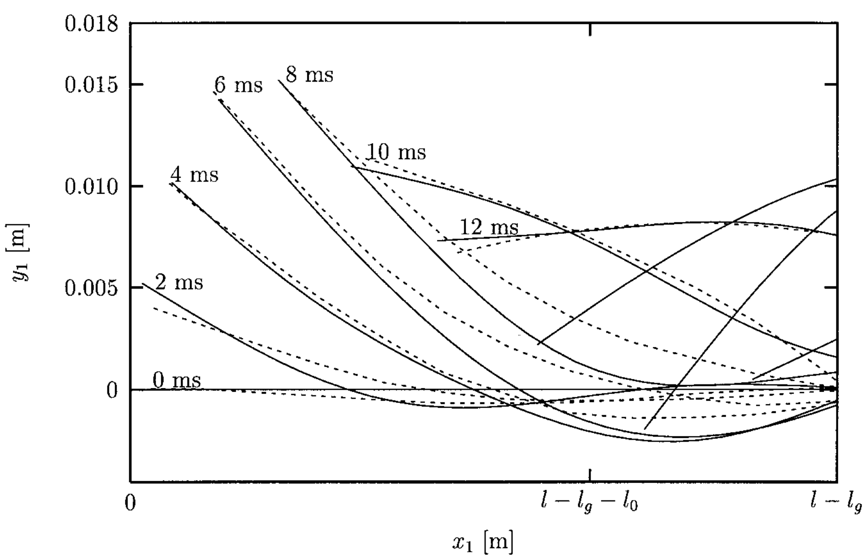

Figure 9 Curves of an arrow's deflection on the basis of experimental data (– – –) (after Pękalski, 1990) and on the basis of the new mathematical model for the arrow (——) every 2 ms after release, and for the model's results until the arrow nock passes the grip. The acceleration force, E, is the predicted force shown in Fig. 6.

In Fig. 10, for both models of the bow, the contact force between the arrow and the grip, denoted by $R(t_1)$, are shown. The model predicts that, shortly after release, the arrow leaves the button again for a very short period – that is, the recoil force is zero during that period. The contact ends at $t_f < t_l$, so before the arrow leaves the string.

The displacement of the arrow nock for both models of the bow is shown in Fig. 11. Let t_g denote the instant that the arrow nock passes the grip. It is important that the nock clears the grip at this moment. After the arrow leaves the string, $t_1 = t_l$, it passes the median plane of the bow ($y_1 = 0$) but before $t_1 = t_g$. This is related to the 'archer's paradox' (Klopsteg, 1992). The arrow does

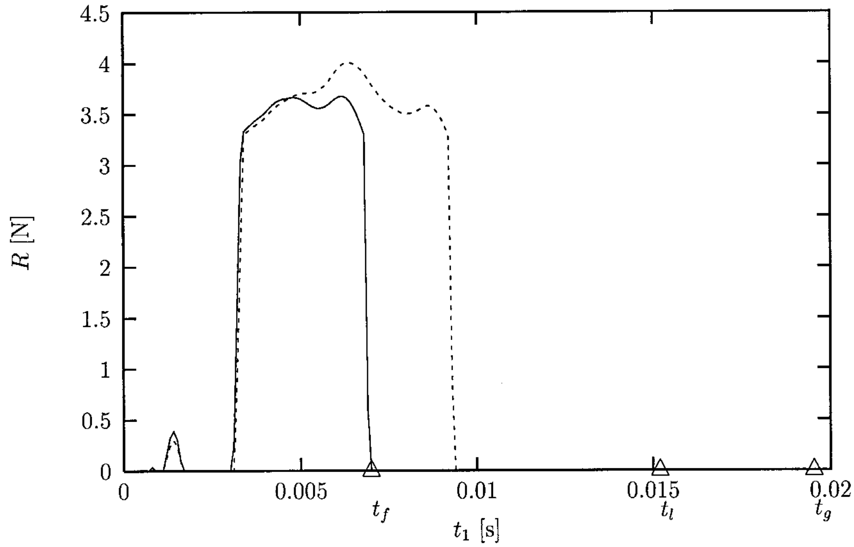

Figure 10 Contact force $R(t_1)$ as a function of time t_1. The solid line (——) is obtained when the acceleration force, E, is the predicted force shown in Fig. 6. The arrow is freed from the grip for a short time before the contact force becomes too large and leaves the grip at time t_f, indicated by the symbol '△' on the t_1-axis. At t_l the arrow leaves the string and at t_g the arrow nock passes the grip. The dashed line (– – –) is obtained when the bow is modelled as a linear spring, where the drawing force, E_n, is proportional to the actual drawing length.

not slap its rear end against the grip but snakes around it; this makes this process central to the shot, because this movement improves the accuracy of the shot. Pękalski did not consider this important feature of the arrow's kinematics.

Discussion and conclusions

Validity of the model

Because of the inertia of the bow limbs, the static force draw and dynamic force draw curves differ significantly (Fig. 6). In this paper, I have shown that it is unwise to simplify the modelling of the dynamic action of a bow–arrow combination using a simple linear spring model for the bow as was done by Pękalski (1987, 1990) for the calculation of the acceleration force. A comparison of the calculated shapes in Figs 1 and 8 shows that, in the new model, the bending of the arrow is greater. This is caused by the initially rather large normal force associated with the acceleration force, which was neglected by Pękalski (1987, 1990). These results are in agreement with the experimental results of Gallozzi *et al.* (1987) and Leonardi *et al.* (1987) with respect to the period of contact between the arrow and the grip. The contact ends before the arrow leaves the string (Fig. 10).

In Pękalski (1990), verification of the mathematical model was performed by comparing two descriptions of the arrow's movement:

- film data of the real archer bow–arrow system; and
- a description resulting from the computer simulation.

A comparison of Figs 1 and 9 suggests that the simulation results obtained with the new model fit the experimental data from a high-speed film better on the bounds of the observed intervals of time ($t = 0$ and $t = t_l$) as well as space variables. This supports Pękalski's statement that the influence of the rest's edge elasticity should also be taken into consideration; however, other improvements in the model were responsible for the model's fit, which verifies the mathematical model.

Use of the model in archery

For a standard arrow with $d = 6.75$ mm ($\frac{17}{64}$ inch), Pękalski's model predicts that the displacement of the nock of the arrow out of the median plane is zero for a relatively long period preceding exit of the arrow at $t_1 = t_l$ (Pękalski, 1990, fig. 8B). His calculations for soft $d = 5.95$ mm ($\frac{15}{64}$ inch) and stiff $d = 8.33$ mm ($\frac{21}{64}$ inch) arrows suggest that there was no such period for these two arrows. On the basis of these results, Pękalski formulated the following definition of a well selected bow–arrow system:

> A well selected bow–arrow sub-system is any system for which the dimensionless parameters of the mathematical model of the arrow's movement during its contact with the bow, have the same values as for the 'standard' system. This means that all well-selected bow–arrow sub-systems should have, after proper rescaling of the axes of the coordinate system and time (x/l, y/b, t/θ), movement that is identical to the standard arrow.

Thus, his definition tries to formulate in words that the jump in the transverse force acting upon the nock at arrow exit is small for a well chosen arrow. The present results, shown in Fig. 11, indicate that the arrow leaves the string approximately at the instant the nock passes the median plane again. Hence, the definition of Pękalski of a well-selected bow–arrow system may still be useful.

Archers select arrows based on bow weight and draw length according to the recommendations (spine charts) of the manufacturer. During the tuning procedure (see Baier *et al.*, 1976), adjustments to the bow and arrow are made that produce the best possible performance of the arrow–bow–archer combination. Several factors influence the performance, including the dimensions of the arrow, type of release, type of 'grip of the bow' and bow parameters (e.g. the brace height, draw length and so on). The model proposed by Kooi (1991) for the bow and, in this paper, for the arrow flight, make it possible to do this tuning by computer simulation. In these models, all parameters have a clear mechanistic interpretation. Therefore, arrow clearance can be predicted.

However, more elaborate experiments have to be performed for each individual archer to measure how the arrow is released over the fingertips and how the bowhand is moved during the release phase. One way to proceed is to define a 'standard release' based on experimental results obtained from the analysis of high-speed film of a top archer-competitor. Sensitivity analysis for this standard case would yield the most important parameters.

Today, video cameras are used as a tool for analysing errors and teaching proper techniques (Sanchez, 1989; Rabska and van Otteren, 1991; Bolnick *et al.*, 1993). For coaching an individual archer-competitor, images from high-speed video-recordings or photography (e.g. 7000 Hz) taken from above the archer have to be digitized. Experiments similar to those of Keast and Elliott (1990), Leroyer *et al.* (1993) and Stuart and Atha (1990) should be undertaken, in which the archery performance is correlated with movements of the extended fingers during release and of the bowhand during the period the arrow is launched from the bow – the most critical period for the shot. Rabska and van Otteren (1991) provide a specification for slow-motion video production. The extracted data form the input for a computer program based on the work presented in this study; the program can be run on a low-cost personal computer. Results from runs with slightly different parameter settings may give insight into the causes of errors and how to improve technique.

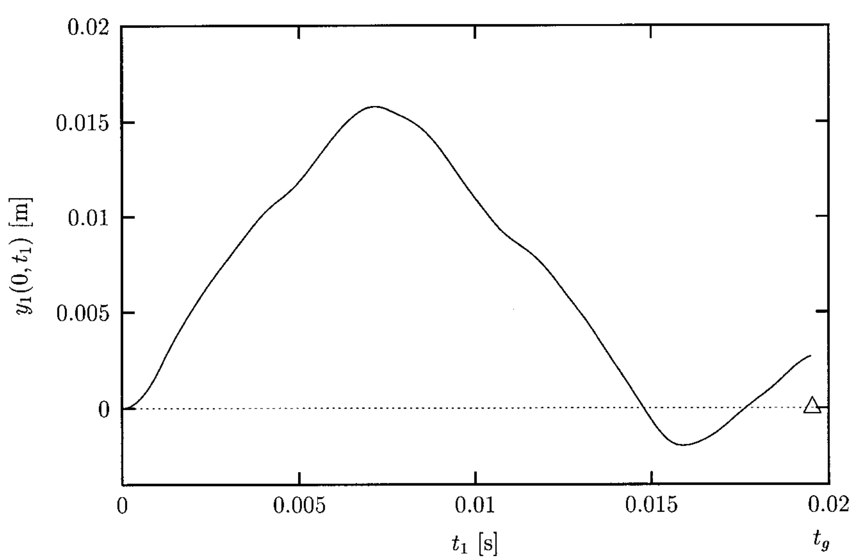

Figure 11 Path of arrow nock for the standard Easton X7 1714 arrow ($d = 6.75$ mm or $\frac{17}{64}$ inch and $g = 0.36$ mm or $\frac{14}{1000}$ inch). The acceleration force, E, is shown in Fig. 6. At t_g, indicated by the symbol '△' on the t_1-axis, the arrow nock passes the grip.

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